Light quark mass effects in the chromomagnetic moment

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Abstract. We present the three-loop QCD corrections to the quark chromomagnetic moment including two different nonzero masses. This is a necessary ingredient to obtain the corresponding corrections to the chromomagnetic coefficient in the Heavy Quark Effective Theory (HQET) Lagrangian.

1. Introduction

The anomalous magnetic moment of the electron and the muon are among the most precisely measured observables in particle physics. Comparing the theoretical and experimental predictions for the muon magnetic moment there is currently a 3.4σ discrepancy [1] with the Standard Model (SM) which makes this observable very interesting at the moment. We calculate finite light quark mass contributions to the chromomagnetic moment of quarks, and obtain, as a byproduct, the corresponding corrections to the above mentioned observables and can confirm the results of Refs. [2, 3, 4]. Another byproduct of our calculation is the anomalous magnetic moment of heavy quarks, the bottom quark in particular, where we include the effect of a finite charm quark mass. The magnetic moment of quarks has not yet been measured experimentally, however, for the bottom and the lighter quarks there are upper limits from LEP1 data [5]. Due to the lack of space in these proceedings we will present analytic results for this observable in Ref. [6].

Whereas the anomalous magnetic moments of fermions are physical observables, the chromomagnetic moment is not. Nevertheless, it plays a crucial role in HQET, where it enters the matching coefficient of the chromomagnetic interaction operator [7]. The one-loop correction to the chromomagnetic moment has been obtained in Refs. [8, 9]. In Refs. [10, 11], the two-loop calculation has been performed, whereas light quark mass effects to this order have been obtained in Ref. [12]. An estimation of higher order corrections has then been given in Ref. [13] and the three-loop correction with one mass scale was finalized in Ref. [7]. In the latter the aforementioned matching coefficient of HQET is almost trivially obtained from the chromomagnetic moment. In the case with two mass scales, additional diagrams have to be calculated in the effective theory to match it to full QCD. We will present the matching coefficient in Ref. [6] and restrict the following discussion to the chromomagnetic moment. The results given in these proceedings have been published in Ref. [14].

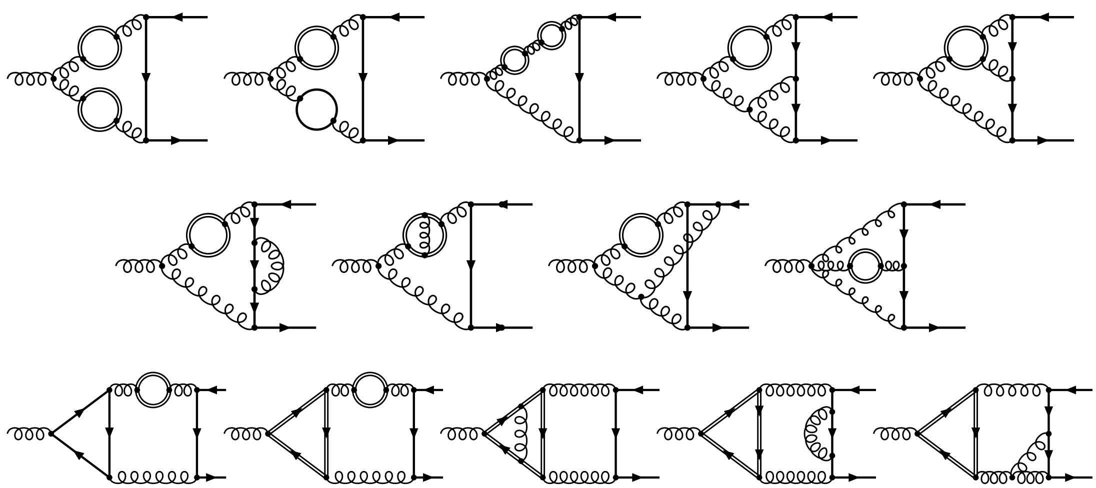

Figure 1. Sample diagrams contributing to the quark chromomagnetic moment. Double solid lines denote light quarks, whereas solid and curly lines denote heavy quarks and gluons, respectively. Note that we use the background field method for the external gluon.

2. Calculation of the chromomagnetic moment

To calculate the chromomagnetic moment we have to consider the quark–anti-quark–gluon vertex in the background-field formalism in QCD. We consider the effect of a nonzero light quark mass at the three-loop level to this quantity. Sample diagrams which have to be calculated are depicted in Fig. 1. When both the quark and anti-quark are on the (renormalised) mass shell and have physical polarisations, the vertex $\Gamma_a^\mu = \Gamma^\mu t_a$ can be decomposed into two form factors,

$$\Gamma^\mu = \gamma^\mu F_1(q^2) - \frac{i}{2m_Q}\sigma^{\mu\nu} q_\nu F_2(q^2) \,, \tag{1}$$

where $q = p_1 - p_2$ is the gluon momentum and p_1 and p_2 are the momenta of the quark and anti-quark, respectively.

The anomalous chromomagnetic moment is given by $\mu_c = Z_2^{\rm OS} F_2(0)$, where $Z_2^{\rm OS}$ is the quark wave function renormalisation constant in the on-shell scheme. The total quark colour charge is given by $Z_2^{\rm OS} F_1(0) = 1$. Thus, $F_1(0)$ is the inverse of the on-shell wave function renormalisation constant, which has been calculated to three-loops including light quark masses in Ref. [15]. Therefore, the calculation of $F_1(0)$ provides a strong check on the correctness of our result.

All Feynman diagrams are generated with `QGRAF` [16] and the various topologies are identified with the help of `q2e` and `exp` [17, 18]. In a next step the reduction of the various functions to so-called master integrals has to be achieved. For this step we use the so-called Laporta method [19, 20] which reduces the three-loop integrals to 27 master integrals. We use the implementation of Laporta's algorithm in the program `Crusher` [21]. It is written in `C++` and uses `GiNaC` [22] for simple manipulations like taking derivatives of polynomial quantities. In the practical implementation of the Laporta algorithm one of the most time-consuming operations is the simplification of the coefficients appearing in front of the individual integrals. This task is performed with the help of `Fermat` [23] where a special interface has been used (see Ref. [24]). The main features of the implementation are the automated generation of the integration-by-parts (IBP) identities [25], a complete symmetrisation of the diagrams and the possibility to make use of a multiprocessor environment. As we need the form factors at zero momentum transfer all occurring master integrals are on-shell propagator-type integrals. They have been

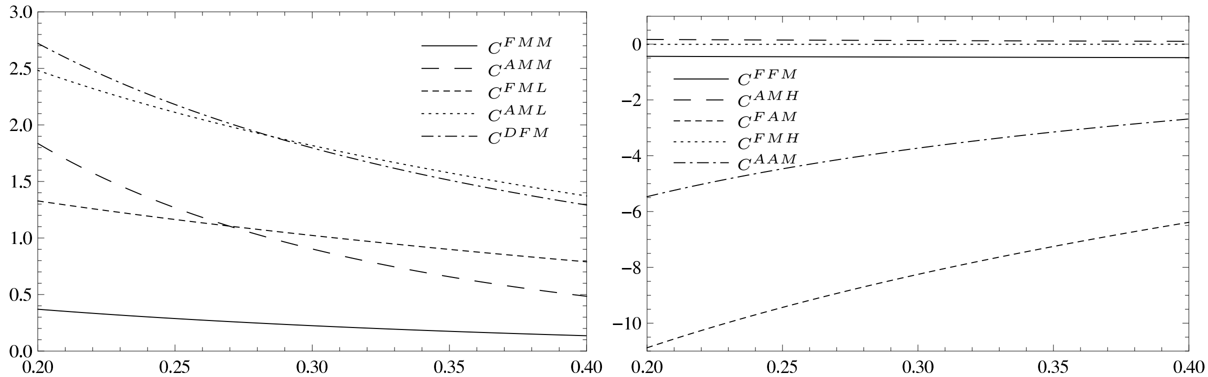

Figure 2. Contributions of $\mathcal{O}(\varepsilon^0)$ from the different colour structures in (3) as a function of the mass ratio x. Note the different scales of the two diagrams.

calculated using different analytical and numerical methods, see Refs. [15, 26] for details. To calculate the colour factors, we have used the program described in Ref. [27].

3. Results

We write the chromomagnetic moment in the form

$$\mu_c(\mu) = 1 + \sum_{n=1}^{\infty} \left(\frac{\alpha_s(\mu)}{\pi}\right)^n \left(\frac{e^{\gamma_E}}{4\pi}\right)^{-n\varepsilon} C^{(n)}, \tag{2}$$

where $\gamma_E = 0,57721...$ is the Euler-Mascheroni constant. α_s denotes the strong coupling constant with $n_f = n_l + n_m + n_h$ active flavours (n_l, n_m and n_h are the number of massless, light and heavy quarks, respectively). In practise, n_m and n_h will be equal to one, but we keep them explicitly in our results in order to track the various classes of diagrams in the result. We further decompose the three-loop contribution containing light quarks with nonzero mass into its colour structures

$$\begin{aligned} C^{(3)}_{n_m} &= C^{FFM} C_F^2 T_F n_m + C^{AAM} C_A^2 T_F n_m + C^{FAM} C_A C_F T_F n_m + C^{FMM} C_F T_F^2 n_m^2 \\ &\quad + C^{FLM} C_F T_F^2 n_l n_m + C^{FMH} C_F T_F^2 n_h n_m + C^{AMM} C_A T_F^2 n_m^2 + C^{ALM} C_A T_F^2 n_l n_m \\ &\quad + C^{AMH} C_A T_F^2 n_h n_m + C^{DFM} \frac{d_F^{abcd} d_F^{abcd} n_m}{C_F N_F}, \end{aligned} \tag{3}$$

where $C_F = (N_c^2 - 1)/(2N_c)$ and $C_A = N_c$ are the eigenvalues of the quadratic Casimir operators of the fundamental and adjoint representation for the SU(N_c) colour group, respectively. In the case of QCD we have $N_c = 3$ and $T_F = 1/2$. The dimension of the fundamental representation is given by $N_F = N_c$. The symmetrised trace of four generators in the fundamental representation is given by $d_F^{abcd} d_F^{abcd} = (N_c^2 - 1)(N_c^4 - 6N_c^2 + 18)/(96N_c^2)$. We present our results at the renormalisation scale $\mu = M_h$, where M_h is the pole mass of the heavy quark. The nonzero pole parts of the various contributions in (3) are given by

$$C^{AMM} = \frac{\ln^2 x}{9\varepsilon} + \mathcal{O}(\varepsilon^0), \tag{4}$$

$$C^{AML} = \frac{\ln x}{18\varepsilon^2} - \frac{1}{\varepsilon}\left(\frac{\ln^2 x}{18} + \frac{13\ln x}{108} + \frac{\pi^2}{432}\right) + \mathcal{O}(\varepsilon^0), \tag{5}$$

$$C^{AAM} = -\frac{\ln x}{9\varepsilon^2} + \frac{1}{\varepsilon}\left(\frac{5\ln^2 x}{72} + \frac{155\ln x}{432} - \frac{\pi^2}{432} - \frac{137}{864} + \frac{\pi^2}{16}x\right. \tag{6}$$

$$+x^2\left(-\frac{\ln^2 x}{16}+\frac{\ln x}{4}-\frac{\pi^2}{96}-\frac{3}{16}\right)-\frac{\pi^2}{96}x^3\Bigg)+\mathcal{O}(\varepsilon^0),$$

$$C^{FAM} = \frac{1}{\varepsilon}\left(\frac{5\ln x}{24}-\frac{235}{576}+\frac{\pi^2}{16}x+x^2\left(\ln x+\frac{3}{4}\right)-\frac{5\pi^2}{16}x^3\right)+\mathcal{O}(\varepsilon^0). \tag{7}$$

The contribution to C^{AAM} and C^{FAM} is presented as a series expansion up to third order in the quark mass ratio $x = M_m/M_h$, where M_m is the pole mass of the light quark. The results for the finite parts of the different colour structures are given in graphical form in Fig. 2 for $0.2 < x < 0.4$, which is relevant for charm mass effects in the chromomagnetic moment of the bottom quark. The mass dependence of the bottom quark in the chromomagnetic moment of the top quark can safely be neglected and the results at $x = 0$ from [7] can be used. The analytic results including the renormalisation scale dependence will be given in [6].

Acknowledgments

This work was supported by the DFG through SFB/TR 9 and the Graduiertenkolleg "Hochenergiephysik und Teilchenastrophysik", the Science and Engineering Research Canada and the Alberta Ingenuity Fund. We would like to thank Jan Piclum for fruitful discussions about the project and Vladimir A. Smirnov for cooperation in the calculation of the diagrams.

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